

# PROBABILISTIC ENERGY MANAGEMENT FOR BUILDING CLIMATE COMFORT IN SMART THERMAL GRIDS WITH SEASONAL STORAGE SYSTEMS

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**ABSTRACT.** This paper presents an energy management framework for building climate comfort systems that are interconnected in a grid via aquifer thermal energy storage (ATES) systems in the presence of two types of uncertainty, namely private and common uncertainty sources. The ATES system is considered as a seasonal storage system that can be a heat source or sink, or a storage for thermal energy. While the private uncertainty source refers to uncertain thermal energy demand of individual buildings, the common uncertainty source describes the uncertain common resource pool (ATES) between neighbors. To this end, we develop a large-scale stochastic hybrid dynamical model to predict the thermal energy imbalance in a network of interconnected building climate comfort systems together with mutual interactions between the local ATES systems. We formulate a finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints at each sampling time, which is in general a non-convex problem and hard to solve. We then provide a computationally tractable framework based on an extension to the so-called robust randomized approach which offers a less conservative solution for a problem with multiple chance constraints. A simulation study is provided to compare three different configurations, namely: completely decoupled, centralized and move-blocking centralized solutions. In addition, we present a numerical study using a geohydrological simulation environment (MODFLOW) to illustrate the advantages of our proposed framework.

**Keywords.** ATES Systems, Smart Thermal Grids, Building Climate Comfort Systems, Multiple Chance Constraints, Probabilistic Robustness, Robust Randomized.

## 1. INTRODUCTION

Global energy consumption has significantly increased due to the combined factors of increasing population and economic growth over the past few decades. This increasing consumption highlights the necessity of employing innovative energy saving technologies. Smart Thermal Grids (STGs) can play an important role in the future of the energy sector by ensuring a heating and cooling supply that is more reliable and affordable for thermal energy networks connecting various households, greenhouses and other buildings, which we refer to as agents. STGs allow for the adaptation to changing circumstances, such as daily, weekly or seasonal variations in supply and demand by facilitating each agent with smart thermal storage technologies.

Aquifer thermal energy storage (ATES) is a less well-known sustainable seasonal storage system that can be used to store large quantities of thermal energy in underground aquifers. It is especially suitable for climate comfort systems of large buildings such as offices, hospitals, universities, musea and greenhouses.

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Most buildings in moderate climates have a heat shortage in winter and a heat surplus in summer. Where aquifers exist, this temporal discrepancy can be overcome by seasonally storing and extracting thermal energy into and out of the subsurface, enabling the reduction of energy usage and CO<sub>2</sub> emissions of climate comfort systems in buildings.

STGs have been studied implicitly in the context of micro combined heat and power systems or general smart grids, e.g., see [1] and [2]. Building heat demand with a dynamical storage tank was considered in [3], whereas in [4] an adaptive-grid model for dynamic simulation of thermocline thermal energy storage systems was developed. A deterministic view on STGs was studied by a few researchers [5], [6]. STGs with uncertain thermal energy demands have been considered in [7], where a model predictive control (MPC) strategy was employed with a heuristic Monte Carlo sampling approach to make the solution robust. A dynamical model of thermal energy imbalance in STGs with a probabilistic view on uncertain thermal energy demands was established in [8], where a stochastic MPC with a theoretical guarantee on the feasibility of the obtained solution was developed.

ATES as a seasonal storage system has not, to the best of our knowledge, been considered in STGs. In [9] and [10], a dynamical model for an ATES system integrated in a building climate comfort system has been developed. Following these studies, the first results toward developing an optimal operational framework to control ATES systems in STGs is presented here. In this framework, uncertain thermal energy demands are considered along with the possible mutual interactions between ATES systems, which may cause limited performance and reduced energy savings. The main contributions of this paper are twofold:

- a) We develop a novel large-scale stochastic hybrid dynamical model to predict the dynamics of thermal energy imbalance in STGs consisting of building climate comfort systems with hourly-based operation and ATES as a seasonal energy storage system. Using an MPC paradigm, we formulate a finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints at each sampling time leading to a non-convex problem, which is difficult to solve.
- b) We develop a computationally tractable framework to approximate a solution for our proposed formulation based on our previous work in [8]. In particular, we extend the framework in [8] to cope with multiple chance constraints which provides a less conservative solution compared to the so-called robust randomized approach in [11]. Our framework is closely related to, albeit different from the approach of [12]. In [12] the problem formulation consists of an objective function with multiple chance constraints, in which the terms in objective and constraints are univariate. In contrast the objective function in our problem formulation consists of separable additive components.

The layout of this paper is as follows: Section 2 describes dynamics of an ATES system and a building climate comfort system. In Section 3, we first formulate an energy management problem in a single agent, and then, extend it to a network of multiple agents. After which, follows three different setups, namely: one with completely decoupled agents, a centralized problem and a move-blocking centralized problem formulation. In Section 4, we develop a computationally tractable framework to solve these problems, and Section 5 provides a simulation study with a comparison between these three different settings. In addition, the numerical results obtained via a geohydrological simulation environment (MODFLOW) are shown. Section 6 concludes this paper with some remarks and future work.

## NOTATION

The following international system of units is widely used: Kelvin [K] and Celsius [°C] are the units of temperature, Meter [m] is the unit of length, Hour [h] is the unit of time, Kilogram [kg] is the unit of mass,

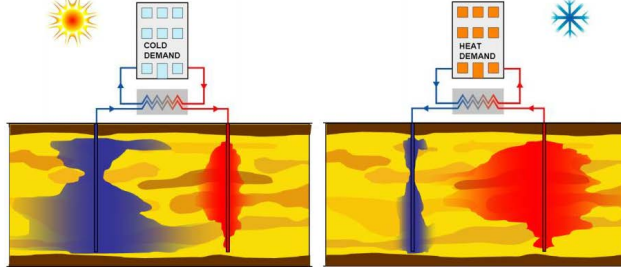


FIGURE 1. Operational modes of an ATES system during warm (left) and cold (right) seasons. Figure is taken from [10].

Watt [W] is the unit of power, Joule [J], KiloWatt-hour [kWh], and MegaWatt-hour [MWh] are the units of energy.

$\mathbb{R}, \mathbb{R}_+$  denote the real and positive real numbers, and  $\mathbb{N}, \mathbb{N}_+$  the natural and positive natural numbers, respectively. We operate within  $n$ -dimensional space  $\mathbb{R}^n$  composed by column vectors  $u, v \in \mathbb{R}^n$ . The Cartesian product over  $n$  sets  $\mathcal{X}_1, \dots, \mathcal{X}_n$  is given by:  $\prod_{i=1}^n \mathcal{X}_i = \mathcal{X}_1 \times \dots \times \mathcal{X}_n = \{(x_1, \dots, x_n) : x_i \in \mathcal{X}_i\}$ . The cardinality of a set  $\mathcal{A}$  is shown by  $|\mathcal{A}| = A$ .

Given a metric space  $\Delta$ , its Borel  $\sigma$ -algebra is denoted by  $\mathfrak{B}(\Delta)$ . Throughout the paper, measurability always refers to Borel measurability. In a probability space  $(\Delta, \mathfrak{B}(\Delta), \mathbb{P})$ , we denote the  $N$ -Cartesian product set of  $\Delta$  by  $\Delta^N$  with the respective product measure by  $\mathbb{P}^N$ .

## 2. SYSTEM DYNAMICS MODELING

In this section, we first develop a mathematical model for an ATES system dynamics as a single seasonal storage system. We then describe the steady-state dynamical model for a building climate comfort system to capture its thermal energy demand profile during heating and cooling modes based on our previous work in [13]. In Section 3, we will use the developments of current section to propose a model dynamics of an ATES system integrated into the building climate comfort system.

### 2.1. Seasonal Storage Systems

We consider an ATES system consisting of warm and cold wells to store warm water during warm season and cold water during cold season, respectively. Each well can be described as a single thermal energy storage where the amount of stored energy is proportional to the temperature difference between stored water and aquifer ambient water. Stored thermal energy from the last season is going to be used for the current season and so forth. Fig. 1 depicts the operating modes of an ATES system for a single building. Depending on the season, the operating mode (heating or cooling) of an ATES system changes, by reversing the direction of water between wells as it is shown in Fig. 1. During a cold season for heating purpose, the direction of water is from the warm well to the cold well through a heat exchanger to extract the stored thermal energy from the water. The return water is cold down to store in the cold well. This procedure is opposite during a warm season for cooling purpose of the building climate comfort system. An ATES system can be characterized by some physically meaningful parameters. The most relevant features that can describe the status of an ATES system for an optimal control purpose is the stored volume of water together with the thermal energy content in each well. A free manipulated variable in this setting is the pump flow rate that is used to circulate water from one well to the other through a heat exchanger.

We therefore define the states that can describe the ATES system dynamics to be the volume of water,  $V_{a,k}^h [\text{m}^3]$ ,  $V_{a,k}^c [\text{m}^3]$ , and the thermal energy content,  $S_{a,k}^h [\text{W}]$ ,  $S_{a,k}^c [\text{W}]$ , of warm and cold wells. Consider the following first-order difference equations as an ATES system model dynamics:

$$V_{a,k+1}^h = V_{a,k}^h - (u_{a,k}^h - u_{a,k}^c), \quad (1a)$$

$$V_{a,k+1}^c = V_{a,k}^c + (u_{a,k}^h - u_{a,k}^c), \quad (1b)$$

$$S_{a,k+1}^h = \eta_a S_{a,k}^h - (h_{a,k}^h - h_{a,k}^c), \quad (1c)$$

$$S_{a,k+1}^c = \eta_a S_{a,k}^c + (h_{a,k}^h - h_{a,k}^c), \quad (1d)$$

where  $\eta_a \in (0, 1)$  is a lumped coefficient of thermal energy losses in aquifers,  $u_{a,k}^h [\text{m}^3\text{h}^{-1}]$ , and  $u_{a,k}^c [\text{m}^3\text{h}^{-1}]$  are control variables corresponding to the pump flow rate of ATES system during heating and cooling modes at each sampling time  $k = 1, 2, \dots$ , receptively.  $u_{a,k}^h$  circulates water from warm well to cold well, whereas  $u_{a,k}^c$  takes water from cold well and injects into warm well of ATES system, during heating modes and cooling modes of the building climate control system, respectively.  $h_{a,k}^h [\text{W}]$ ,  $c_{a,k}^h [\text{W}]$  denote the amount of thermal energy that is extracted from warm well and injected into cold well of ATES system during heating mode of building climate comfort system, respectively.  $c_{a,k}^c [\text{W}]$ ,  $h_{a,k}^c [\text{W}]$  are the amount of thermal energy that is extracted from cold well and injected into warm well of ATES system during cooling mode of building climate comfort system, respectively. They are defined by:

$$\begin{cases} h_{a,k}^h = \alpha_h u_{a,k}^h \\ c_{a,k}^h = \alpha_c u_{a,k}^h \end{cases}, \quad \begin{cases} c_{a,k}^c = \alpha_c u_{a,k}^c \\ h_{a,k}^c = \alpha_h u_{a,k}^c \end{cases},$$

where  $\alpha_h = \rho_w c_{pw} (T_{a,k}^h - T_{a,k}^{\text{amb}})$ , and  $\alpha_c = \rho_w c_{pw} (T_{a,k}^{\text{amb}} - T_{a,k}^c)$  are the thermal energy coefficients of warm and cold wells, respectively.  $\rho_w [\text{kgm}^{-3}]$ ,  $c_{pw} [\text{Jkg}^{-1}\text{K}^{-1}]$  are density and specific heat capacity of water, respectively.  $T_{a,k}^h [\text{K}]$ ,  $T_{a,k}^c [\text{K}]$ , and  $T_{a,k}^{\text{amb}} [\text{K}]$  denote the temperature of water inside warm well, cold well and the aquifer ambient, respectively. We also define  $h_{a,k} [\text{W}]$ , and  $c_{a,k} [\text{W}]$  to be the amount of thermal energy that can be delivered to the building during heating and cooling modes, respectively, as follows:

$$\begin{cases} h_{a,k} = \alpha u_{a,k}^h \\ c_{a,k} = \alpha u_{a,k}^c \end{cases},$$

where  $\alpha = \alpha_h + \alpha_c$  is the total thermal energy coefficient. It is worth to note that we also develop another control-oriented model framework for the integrated ATES system model into the building climate comfort system in [9], where we consider to have dynamical behavior of the volume and temperature of water in each well of ATES system.

Let us now discuss the proposed dynamics of ATES system in (1). Equations (1a), (1b), (1c), and (1d) describe the evolution of water volume and the thermal energy content in warm and cold wells, respectively. During cold seasons for the heating purpose of building climate comfort system, the amount of  $u_{a,k}^h$  volume of warm water from the warm well is extracted to provide  $h_{a,k}$  amount thermal energy, and meanwhile, the amount of  $c_{a,k}^h$  thermal energy is stored in the cold well of ATES system. As for the cooling purpose of building climate comfort system during warm seasons, the amount of  $u_{a,k}^c$  volume of cold water from the cold well is extracted to provide  $c_{a,k}$  amount thermal energy, while the amount of  $h_{a,k}^c$  thermal energy is injected in the warm well of ATES system. Dynamics of ATES systems are visualized in Fig. 2, which represents the relation between the state variables of ATES system. Operating modes during cold and warm seasons are shown via red color and blue color, respectively. The developed model (1) is a discrete-time linear system.

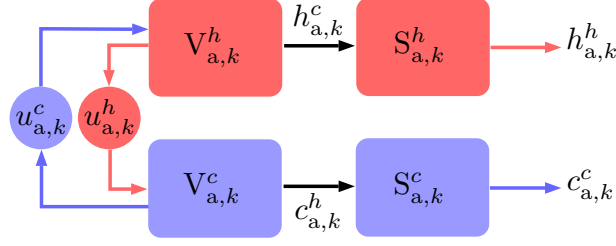


FIGURE 2. Block diagram representation of ATES system showing the relation between an ATES system input, state and output variables. Operating modes during cold and warm seasons are shown via red color and blue color, respectively.

**Remark 1.** *There is always only one operating mode active in ATES systems, which leads to have only one of control variables to be nonzero:*

$$u_{a,k}^h u_{a,k}^c = 0, \quad k = 1, 2, \dots$$

## 2.2. Building Climate Comfort Systems

In our previous work [13], a dynamical model of a building climate comfort system was developed to determine the thermal energy demand of building at each sampling time  $k$ , considering the desired indoor air temperature and the outside weather conditions of building. We refer to the building climate comfort system that determines the level of thermal energy demand  $Q_{d,k}^B$  [W] at each sampling time  $k$  via

$$Q_{d,k}^B = f_B(p_s^B, T_{des,k}^B, \vartheta_k), \quad (2)$$

where  $p_s^B, T_{des,k}^B$  [°C] denote a parameter vector and a desired indoor air temperature of building, respectively.  $\vartheta_k = [T_{o,k}^B, I_{o,k}, v_{o,k}, Q_{p,k}, Q_{e,k}] \in \mathbb{R}^5$  is a vector of uncertain variables that contains outside air temperature, solar radiation, wind velocity, the thermal energy produced due to occupancy by people and total electrical devices/lighting installation inside the building. This yields the building thermal energy demand that takes into account the overall building effects, e.g. zones, walls, humans and non-human thermal energy sources together with the outside uncertain weather conditions.

**Remark 2.** *We are interested in capturing the variation of thermal energy demand w.r.t. the outside air temperature  $T_{o,k}^B$ . Therefore, the uncertain variable in (2),  $\vartheta_k$ , is assigned to  $T_{o,k}^B$ , and the rest of the variables are fixed to their nominal (forecast) values at each sampling time  $k$ . From (2), it follows that the mapping from the uncertain variable  $\vartheta_k$  to the thermal energy demand  $Q_{d,k}^B$  is measurable, so that  $Q_{d,k}^B$  can be viewed as a random variable on the same probability space as  $\vartheta_k$ .*

**Remark 3.** *The operating modes (heating or cooling) of building climate comfort system are determined based on the sign of  $Q_{d,k}^B$  at each sampling time  $k$ .  $Q_{d,k}^B$  with positive and negative signs, represents the thermal energy demand during heating mode and the building surplus thermal energy during cooling mode, respectively.  $Q_{d,k}^B = 0$ , is related to the comfort mode of building, and thus, no heating or cooling is requested. We also distinguish between the thermal energy demand of building during heating mode  $h_{d,k}$ , and cooling mode  $c_{d,k}$ , using the relation:  $Q_{d,k}^B = h_{d,k} - c_{d,k}$ . Moreover, the thermal energy demand at each sampling time  $k$  can be only either for heating  $h_{d,k}$ , or cooling  $c_{d,k}$  modes, which leads to have*

$$h_{d,k} c_{d,k} = 0, \quad k = 1, 2, \dots$$

Fig. 3 shows the thermal energy demand profile of a building for the last five years with respect to the outside registered weather data of Delft, the Netherlands. The building parameters are considered to be as

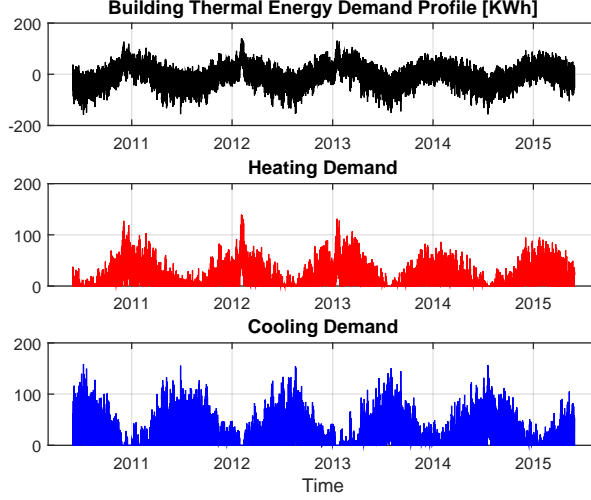


FIGURE 3. Thermal energy demand profile of a building for the last five years with respect to the outside registered weather data of Delft, the Netherlands. The building parameters are considered to be as the faculty of 3ME of TU Delft. The black line shows  $Q_{d,k}^B$ , the red line is related to the thermal energy demand during heating mode  $h_{d,k}$  and the blue line corresponds to the thermal energy surplus during cooling mode  $c_{d,k}$ .

the faculty of Mechanical, Maritime and Materials Engineering (3ME) of Delft University of Technology (TU Delft). The top panel in Fig. 3 depicts  $Q_{d,k}^B$  as the result of (2), whereas the middle and down panels show the thermal energy demand during heating mode  $h_{d,k}$  and the thermal energy surplus during cooling mode  $c_{d,k}$ , respectively.

### 3. ENERGY MANAGEMENT PROBLEM

In this section, we formulate an optimization problem for heating and cooling modes of the building climate comfort system which represents a single agent energy management problem. We describe the equipment of building during heating and cooling modes, taking into account the developed model of ATES system in Section 2. We then extend a single agent problem formulation to a networks with multiple agents that can be producers and consumers of thermal energy in a smart grid setting. The goal of the agents is to match the local consumption and production to avoid mutual interactions between their ATES systems in the network and improve energy efficiency.

#### 3.1. Energy Balance in Single Agent System

Consider a single agent (i.e. building)  $i \in \{1, \dots, N\}$  that is facilitated with a boiler, a heat pump, a storage tank for the heating mode, and a chiller, a storage tank for the cooling mode together with an ATES system that is available for both operating modes (see Fig. 4). For a day-ahead planning problem of each agent, we consider a finite-horizon  $N_h$  with hourly steps, and introduce the subscript  $t$  in our notation to characterize the value of the quantities for a given time instance  $t \in \{k, k+1, \dots, N_h+k\}$ .

For each agent  $i$  one can rewrite the proposed dynamics of ATES system in (1) in a more compact format:

$$x_{i,t+1}^a = a_i^a x_{i,t}^a + b_i^a u_{i,t}^a, \quad (3)$$

where  $x_{i,t}^a = \begin{bmatrix} V_{a,t}^h & V_{a,t}^c & S_{a,t}^h & S_{a,t}^c \end{bmatrix}^\top \in \mathbb{R}^4$  denotes the state vector,  $u_{i,t}^a = \begin{bmatrix} u_{a,t}^h & u_{a,t}^c \end{bmatrix}^\top \in \mathbb{R}^2$  is the control vector, and  $a_i^a, b_i^a$  can be obtained via (1), as follows:

$$a_i^a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \eta_a & 0 \\ 0 & 0 & 0 & \eta_a \end{bmatrix}, \quad b_i^a = \begin{bmatrix} -1 & 1 \\ 1 & -1 \\ -\alpha_h & \alpha_h \\ \alpha_c & -\alpha_c \end{bmatrix}.$$

An important operational limitation of ATEs systems is that the sum of injected and extracted thermal energy over a specific period of time (typically a year),  $N_y$ , has to be zero:

$$\sum_{t=k}^{N_y} (h_{a,t}^h - h_{a,t}^c) = 0, \quad \sum_{t=k}^{N_y} (c_{a,t}^c - c_{a,t}^h) = 0. \quad (4)$$

This restriction imposed by the government to prevent any long-term changes/effects in the aquifer ambient temperature and to make the ATEs system sustainable [14]. These constraints should be satisfied within one calendar year or longer periods of time (once in each five years). To handle such a constraint, one can use shrinking-horizon dynamic programming [15]. In our proposed model (1), the amount of thermal energy content in each well is defined to be the state variable of an ATEs system. This yields an advantage to reformulate (4) as follows:

$$S_{a,t}^h + S_{a,t}^c = \bar{S}_a, \quad (5)$$

where  $\bar{S}_a$  corresponds to the initial amount of thermal energy which exists in the wells of ATEs system. Notice that the proposed constraint (5) is the same as the constraint (4) with slightly more restrictions. We pose the equality of the extraction and injection of thermal energy over a finite time steps, whereas in constraint (5) we restrict the equality of the extraction and injection of the thermal energy at each sampling time, and therefore, this would lead to a more conservative solution. To this end, we introduce an auxiliary control variable  $e_{i,t}$  for each agent  $i$  at each sampling time  $t$  to soften the formulated constraint (5). We now impose the following constraints

$$S_{a,t}^h + S_{a,t}^c \leq \bar{S}_a + e_{i,t}, \quad S_{a,t}^h + S_{a,t}^c \geq \bar{S}_a - e_{i,t}. \quad (6)$$

It is important to mention that the proposed reformulation (6) is not meant to be an equivalent constraint as (4). This is due to the fact that (4) has to be satisfied within a longer period of time, whereas (6) is imposed along the prediction horizon. We however state here that (6) may be equivalent with (4), whenever the prediction horizon is long enough (a year) and it is imposed only at the final step.

We now focus on the energy balance formulation of the building climate comfort system. In spite of our previous work in [8], we here define two vectors of control variables during heating and cooling modes in each agent  $i$  at each sampling time  $t$ , to be

$$u_{i,t}^h = \begin{bmatrix} h_{\text{boi},t} & h_{\text{im},t} \end{bmatrix}^\top \in \mathbb{R}^2, \quad u_{i,t}^c = \begin{bmatrix} c_{\text{chi},t} & c_{\text{im},t} \end{bmatrix}^\top \in \mathbb{R}^2.$$

$h_{\text{boi},t}$ ,  $c_{\text{chi},t}$ ,  $h_{\text{im},t}$ , and  $c_{\text{im},t}$  denote the production of boiler, chiller, the imported energies from external parties during heating and cooling modes, respectively. We also consider to have freedom to decide about the on-off status of boiler and chiller by  $v_{i,t} = \begin{bmatrix} v_{\text{boi},t} & v_{\text{chi},t} \end{bmatrix}^\top \in \{0,1\}^2$ . Moreover, the startup cost of boiler and chiller are taken into account by  $c_{i,t}^{\text{su}} = \begin{bmatrix} c_{\text{boi},t}^{\text{su}} & c_{\text{chi},t}^{\text{su}} \end{bmatrix}^\top \in \mathbb{R}^2$  for each agent  $i$  at each time step  $t$ . We define two variables to capture the thermal energy imbalance errors during heating mode  $x_{i,t}^h \in \mathbb{R}$ , and an imbalance error of the cooling mode  $x_{i,t}^c \in \mathbb{R}$ . They are related to the difference between the level of storage

tank with the forecast thermal energy demand,  $h_{d,t}^f$ ,  $c_{d,t}^f$ , during heating and cold modes, respectively. They are formally defined as

$$x_{i,t}^h = h_{s,t} - h_{d,t}^f, \quad (7a)$$

$$x_{i,t}^c = c_{s,t} - c_{d,t}^f, \quad (7b)$$

where  $h_{s,t}$ , and  $c_{s,t}$  represent the level of storage tank during heating and cooling modes, respectively. They have the following dynamics:

$$h_{s,t+1} = \eta_s^h x_{i,t}^h + \eta_s^h (h_{\text{boi},t} + h_{\text{im},t} + \alpha_{\text{hp}} h_{a,t}),$$

$$c_{s,t+1} = \eta_s^c x_{i,t}^c + \eta_s^c (c_{\text{chi},t} + c_{\text{im},t} + c_{a,t}),$$

where  $\alpha_{\text{hp}} = \text{COP}(\text{COP} - 1)^{-1}$  is related to the effect of heat pump,  $\eta_s^h \in (0, 1)$ , and  $\eta_s^c \in (0, 1)$  denote the thermal loss coefficients due to inefficiency of storage tank storage during heating and cooling modes, respectively. COP stands for the coefficient of performance of heat pump.  $h_{a,t}$  and  $c_{a,t}$  present the amount thermal energy that is extracted from warm and cold wells of ATES system for heating and cooling modes, respectively. It is important to notice that  $h_{a,t}$  and  $c_{a,t}$  are dependent on the pump flow rates  $u_{a,t}^h$  and  $u_{a,t}^c$  of the ATES system during heating and cooling modes of the building climate comfort system, respectively. We now substitute  $h_{s,t}$ , and  $c_{s,t}$  in (7) to derive the dynamical behavior of the thermal energy imbalance  $x_{i,t}^h$  and  $x_{i,t}^c$  that are given by

$$x_{i,t+1}^h = a_i^h x_{i,t}^h + b_i^h u_{i,t}^h + b_{i,a}^h u_{i,t}^a + c_i^h w_{i,t}^h, \quad (8a)$$

$$x_{i,t+1}^c = a_i^c x_{i,t}^c + b_i^c u_{i,t}^c + b_{i,a}^c u_{i,t}^a + c_i^c w_{i,t}^c, \quad (8b)$$

where  $a_i^h = \eta_s^h$ ,  $a_i^c = \eta_s^c$ ,  $b_i^h = \begin{bmatrix} \eta_s^h & \eta_s^h \end{bmatrix} \in \mathbb{R}^{1 \times 2}$ ,  $b_i^c = \begin{bmatrix} \eta_s^c & \eta_s^c \end{bmatrix} \in \mathbb{R}^{1 \times 2}$ ,  $b_{i,a}^h = \begin{bmatrix} \eta_s^h \alpha_{\text{hp}} & 0 \end{bmatrix} \in \mathbb{R}^{1 \times 2}$ ,  $b_{i,a}^c = \begin{bmatrix} \eta_s^c \alpha & 0 \end{bmatrix} \in \mathbb{R}^{1 \times 2}$ ,  $c_i^h = -1$ , and  $c_i^c = -1$ . The variables  $w_{i,t}^h = h_{d,t+1}^f$  and  $w_{i,t}^c = c_{d,t+1}^f$  refer to the forecast of thermal energy demand during heating and cooling modes in the next time step, respectively. The only uncertain variable in each agent  $i$  at each sampling time  $t$  is considered to be the deviation of actual thermal energy demand from its forecast value, and therefore,  $w_{i,t}^h$  and  $w_{i,t}^c$  represent uncertain parameters. Consider now the system dynamics for each agent  $i$  by concatenating the thermal energy imbalance errors during heating and cooling modes together with the state vector of the ATES system as follows:

$$x_{i,t+1} = a_i x_{i,t} + b_i u_{i,t} + c_i w_{i,t}, \quad (9)$$

where  $a_i, b_i, c_i$  are system parameters,  $x_{i,t} = \begin{bmatrix} x_{i,t}^h & x_{i,t}^c & x_{i,t}^a \end{bmatrix}^\top \in \mathbb{R}^6$  denotes the state vector,  $u_{i,t} = \begin{bmatrix} u_{i,t}^h & u_{i,t}^c & u_{i,t}^a & c_{i,t}^{\text{su}} & e_{i,t} \end{bmatrix}^\top \in \mathbb{R}^9$  is the control vector, and  $w_{i,t} = \begin{bmatrix} w_{i,t}^h & w_{i,t}^c \end{bmatrix}^\top \in \mathbb{R}^2$  is the uncertainty vector.  $a_i \in \mathbb{R}^{6 \times 6}$ ,  $b_i \in \mathbb{R}^{6 \times 9}$ , and  $c_i \in \mathbb{R}^{6 \times 2}$  are as follows:

$$a_i = \begin{bmatrix} a_{i,t}^h & 0 & \mathbf{0}_{1 \times 4} \\ 0 & a_{i,t}^c & \mathbf{0}_{1 \times 4} \\ 0 & 0 & a_{i,t}^a \end{bmatrix}, \quad b_i = \begin{bmatrix} b_i^h & \mathbf{0}_{1 \times 2} & b_{i,a}^h & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{1 \times 2} & b_{i,t}^c & b_{i,a}^c & \mathbf{0}_{1 \times 3} \\ \mathbf{0}_{4 \times 2} & \mathbf{0}_{4 \times 2} & b_i^a & \mathbf{0}_{4 \times 3} \end{bmatrix}, \quad c_i = \begin{bmatrix} c_{i,t}^h & 0 \\ 0 & c_{i,t}^c \\ \mathbf{0}_{4 \times 1} & \mathbf{0}_{4 \times 1} \end{bmatrix}.$$

The objective function in each agent  $i$  is to map local the thermal energy supply of production unit to the local thermal energy demand of building climate comfort system. Our goal therefore is to find the control input  $u_{i,t}$  for each agent  $i$  such that the thermal energy imbalance errors stay as a small as possible at minimal production cost and to satisfy physical constraints together with the minimization of energy balance error  $e_{i,t}$  of ATES system in each agent  $i$  at each sampling time  $t$ . We associate a quadratic cost function with each agent  $i$  at each time step  $t$  as follows:

$$J_i(x_{i,t}, u_{i,t}) = x_{i,t}^\top Q_i x_{i,t} + u_{i,t}^\top R_i u_{i,t}, \quad (10)$$



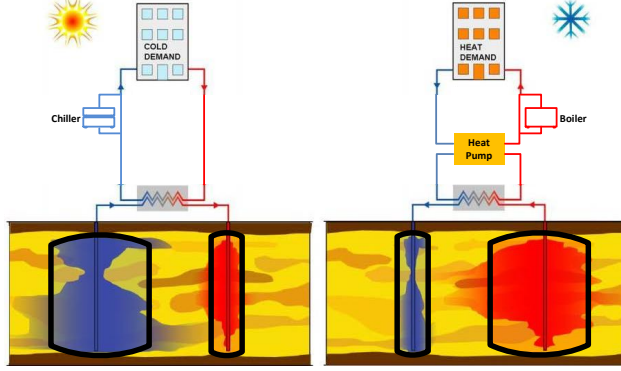


FIGURE 4. Heating and cooling operating modes of building climate comfort system with an ATEs system during warm (left) and cold (right) seasons.

where  $Q_i = \text{diag} \left( \begin{bmatrix} q_i^h & q_i^c & \mathbf{0}_{1 \times 4} \end{bmatrix} \right) \in \mathbb{R}^{6 \times 6}$ ,  $R_i = \text{diag} (r_i) \in \mathbb{R}^{9 \times 9}$  indicate two diagonal matrices with the weighting coefficients of thermal energy imbalance errors and the cost vector  $r_i$  on their diagonal, respectively.  $r_i$  is defined as

$$r_i = \left[ r_{\text{boi}} \quad r_{\text{im}}^h \quad r_{\text{chi}} \quad r_{\text{im}}^c \quad r_a^h \quad r_a^c \quad 1 \quad 1 \quad 1 \right]^\top \in \mathbb{R}^9 ,$$

where  $r_{\text{boi}}$  ( $r_{\text{chi}}$ ) relates to the cost of natural gas that is used by boiler (chiller),  $r_{\text{im}}^h$  ( $r_{\text{im}}^c$ ) denotes to the cost of imported thermal energy from an external party during heating (cooling) mode, and  $r_a^h$  ( $r_a^c$ ) corresponds to the electricity cost of pump of ATEs system to extract the required thermal energy during heating (cooling) modes. The other entries of  $r_i$  represent the start-up costs together with a coefficient for the auxiliary variable  $e_{i,t}$ . The proposed cost function consists of three main parts which leads to the regulation of imbalance errors to zero at minimal production cost together with minimum energy balance error of ATEs system in each agent  $i$ . The reason for introducing cost function in this form is that from computational perspective quadratic cost functions are motivated by convexity and differentiability arguments.

**Remark 4.** The cost function  $J_i(\cdot)$  is a random variable, and thus, we consider  $\mathbb{E} [J_i(\cdot)]$  to obtain a deterministic cost function.

We are now in a position to formulate the following optimization problem for each agent  $i = 1, 2, \dots, N$  at each sampling time  $t$ :

$$\begin{aligned} & \underset{\{u_{i,t}, v_{i,t}\}_{t=k}^{k+N_h}}{\text{minimize}} && \sum_{t=k}^{k+N_h} \mathbb{E} [J_i(x_{i,t}, u_{i,t})] \end{aligned} \quad (11a)$$

$$\text{subject to} \quad c_{i,t}^{su} \geq \Lambda^{su} (v_{i,t} - v_{i,t-1}) , \quad c_{i,t}^{su} \geq 0 , \quad (11b)$$

$$v_{\text{boi},t} h_{\text{boi}}^{\min} \leq h_{\text{boi},t} \leq h_{\text{boi}}^{\max} v_{\text{boi},t} , \quad (11c)$$

$$v_{\text{chi},t} c_{\text{chi}}^{\min} \leq c_{\text{chi},t} \leq c_{\text{chi}}^{\max} v_{\text{chi},t} , \quad (11d)$$

$$h_{\text{im}}^{\min} \leq h_{\text{im},t} \leq h_{\text{im}}^{\max} , \quad (11e)$$

$$c_{\text{im}}^{\min} \leq c_{\text{im},t} \leq c_{\text{im}}^{\max} , \quad (11f)$$

$$u_a^{\min} \leq u_{a,t}^h \leq u_a^{\max} , \quad (11g)$$

$$u_a^{\min} \leq u_{a,t}^c \leq u_a^{\max} , \quad (11h)$$

$$S_{a,t}^h + S_{a,t}^c \leq \bar{S}_a + e_{i,t} , \quad (11i)$$

$$\mathbb{P} \{ x_{i,t+1} \geq 0 , \quad t = k, \dots, k + N_h \} \geq 1 - \epsilon , \quad (11j)$$

where  $\Lambda^{su}$  is a diagonal matrix including the startup costs of boiler and chiller on the diagonal,  $h_{\text{boi}}^{\min}, h_{\text{boi}}^{\max}, c_{\text{chi}}^{\min}, c_{\text{chi}}^{\max}$  denote the minimum and maximum capacity of thermal energy production of boiler and chiller, respectively,  $h_{\text{im}}^{\min}, h_{\text{im}}^{\max}, c_{\text{im}}^{\min}, c_{\text{im}}^{\max}$  are the minimum and maximum capacity of thermal energy production of each external party during heating and cooling modes, respectively,  $u_a^{\min}, u_a^{\max}$  represent the minimum and maximum pump flow rate of ATES system, respectively, and  $\epsilon \in (0, 1)$  is the admissible constraint violation parameter.

In order of appearance, the constraints have the following meaning. Constraint (11b) captures the status change of boiler and chiller (from off to on). Note that the status change from on to off never appears in the cost function due to the positivity constraint of  $c_{i,t}^{su} \geq 0$ . (11c), (11d), (11e), (11f), (11g), (11h) impose box constraints (capacity limitations) on their variables. In the given lower and upper bounds of both constraints (11c) and (11d), there are multiplications with binary variable which enforce the status change of boiler and chiller, respectively. Constraint (11i) relates to the energy balance error of ATES system as it is explained in (4) and (6). Constraint (11j) ensures probabilistically to have feasible trajectories of the thermal energy imbalance errors for in each agent w.r.t the all possible realization of the uncertain variables  $w_{i,t}^h$  and  $w_{i,t}^c$ . The proposed optimization problem (11) is a finite-horizon, chance-constrained mixed-integer quadratic program, whose stages are coupled by the binaries (11b), and dynamics of the imbalance error (11j) for each agent  $i$  in each sampling time  $k$ . It is important to note that this constraint formulation leads to a non-convex optimization problem and hard to solve in general.

To extend the proposed formulation (11) into the energy management problem of smart thermal grids that will be presented at the end of the preceding section, we first need to introduce some vectors  $\mathbf{x}_i \in \mathbb{R}^{6N_h=n_x} \in \mathbb{R}^{9N_h=n_u}$ ,  $\mathbf{u}_i, \mathbf{v}_i \in \mathbb{R}^{2N_h=n_v}$ , and  $\mathbf{w}_i \in \mathbb{R}^{2N_h=n_w}$ :

$$\begin{aligned} \mathbf{x}_i &= \begin{bmatrix} x_{i,k+1}^\top & x_{i,k+2}^\top & \cdots & x_{i,k+N_h+1}^\top \end{bmatrix}^\top, \quad \mathbf{u}_i = \begin{bmatrix} u_{i,k}^\top & u_{i,k+1}^\top & \cdots & u_{i,k+N_h}^\top \end{bmatrix}^\top, \\ \mathbf{v}_i &= \begin{bmatrix} v_{i,k}^\top & v_{i,k+1}^\top & \cdots & v_{i,k+N_h}^\top \end{bmatrix}^\top, \quad \mathbf{w}_i = \begin{bmatrix} w_{i,k}^\top & w_{i,k+1}^\top & \cdots & w_{i,k+N_h}^\top \end{bmatrix}^\top. \end{aligned}$$

Given the initial value of the state  $x_{i,k}$ , one can eliminate the state variables from the dynamics (9), one can rewrite the system dynamics of each agent  $i$  as

$$\mathbf{x}_i = A_i \mathbf{x}_{i,k} + B_i \mathbf{u}_i + C_i \mathbf{w}_i. \quad (12)$$

where  $\mathbf{x}_i$ ,  $\mathbf{u}_i$ ,  $\mathbf{v}_i$ , and  $\mathbf{w}_i$  represent the vectors of state, control input, binary variables, uncertainty vector that corresponds to a realization (scenario) of the uncertainty over a finite-horizon for agent  $i$ , respectively. The exact form of  $A_i$ ,  $B_i$  and  $C_i$  matrices are omitted in the interest of space and can be found in [16, Section 9.5]. The total cost function  $\mathcal{J}_i(\mathbf{x}_i, \mathbf{u}_i)$  for a finite-horizon prediction at each sampling time  $t$  is given by

$$\mathcal{J}_i(\mathbf{x}_i, \mathbf{u}_i) = \mathbf{x}_i^\top \mathbf{Q}_i \mathbf{x}_i + \mathbf{u}_i^\top \mathbf{R}_i \mathbf{u}_i,$$

where  $\mathbf{Q}_i$  and  $\mathbf{R}_i$  are two block diagonal matrices with  $Q_i$  and  $R_i$  on the diagonal for each agent  $i$ . Note that the sum  $\sum(\cdot)$  and the expected  $\mathbb{E}[\cdot]$  in the cost function (11a) are linear operators and thus, we can change their order without loss of generality. We are now able to formulate a finite-horizon chance-constrained mixed-integer quadratic optimization problem for each agent  $i = 1, \dots, N$ , in a condensed format:

$$\min_{\mathbf{u}_i, \mathbf{v}_i} \quad \mathcal{V}_i(\mathbf{x}_i, \mathbf{u}_i) = \mathbb{E}_{\mathbf{w}_i} [\mathcal{J}_i(\mathbf{x}_i, \mathbf{u}_i)] \quad (13a)$$

$$\text{s.t.} \quad E_i \mathbf{u}_i + F_i \mathbf{v}_i + P_i \leq 0, \quad (13b)$$

$$\mathbb{P}_{\mathbf{w}_i} [A_i \mathbf{x}_{i,k} + B_i \mathbf{u}_i + C_i \mathbf{w}_i \geq 0] \geq 1 - \varepsilon_i, \quad \forall \mathbf{w}_i \in \mathcal{W}_i, \quad (13c)$$

where  $E_i$ ,  $F_i$ ,  $P_i$  are matrices that are built by concatenating all constraints, and  $\varepsilon_i \in (0, 1)$  is the admissible constraint violation parameter.

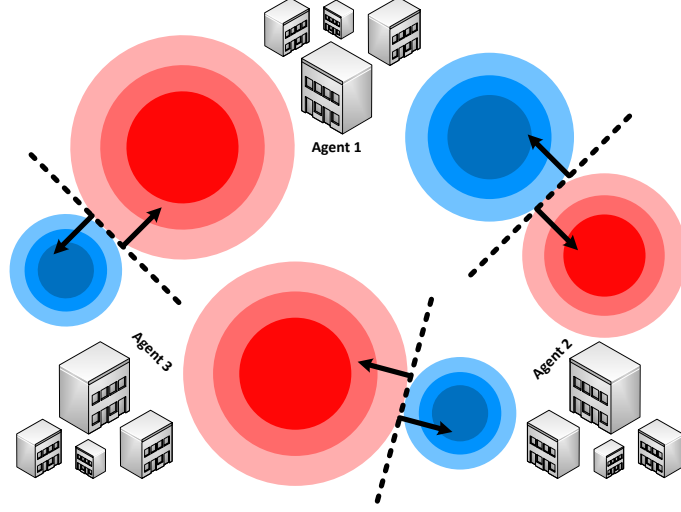


FIGURE 5. Three-agent ATES system in a STG. Each agent has a single ATES system which consists of a warm and a cold well. Horizontal cross sections of warm and cold wells are shown with red and blue circles. The black dashed lines represent the unwanted mutual interactions between neighboring ATES systems.

**Assumption 1.** Following Remark 2,  $\mathbf{w}_i$ , is defined on some probability space  $(\mathcal{W}_i, \mathfrak{B}(\mathcal{W}_i), \mathbb{P}_{\mathbf{w}_i})$ , where  $\mathcal{W}_i \subseteq \mathbb{R}^{n_w}$ ,  $\mathfrak{B}(\cdot)$  denotes a Borel  $\sigma$ -algebra, and  $\mathbb{P}_{\mathbf{w}_i}$  is a probability measure defined over  $\mathcal{W}_i$ .

**Remark 5.** The index of  $\mathbb{E}_{\mathbf{w}_i}, \mathbb{P}_{\mathbf{w}_i}$  denotes the dependency of the state trajectory  $\mathbf{x}_i$  on the string of random scenarios  $\mathbf{w}_i$  for each agent  $i$ . It is worth to mention that for our study we only need a finite number of instances of  $\mathbf{w}_i$ , and we do not require the probability space  $\mathcal{W}_i$  and the probability measure  $\mathbb{P}_{\mathbf{w}_i}$  to be known explicitly. The availability of the number of scenarios from the sample space  $\mathcal{W}_i$  is enough which can for instance be obtained from historical data.

We refer to the proposed optimization problem (13) as a single agent optimization problem, and whenever all agents solve this problem separately in a receding horizon fashion without any coupling constraints, it is referred to as the *decoupled solution* (DS) in the subsequent parts. It is important to notice that the proposed problem (13) is in general a non-convex problem and hard to solve. In the following section, we will develop a tractable framework to obtain an  $\varepsilon_i$ -feasible solution for each agent  $i$ . In the following part, we extend the proposed single agent optimization problem (13) into a multi-agent network (STG).

### 3.2. ATES Smart Thermal Grids

Consider a regional thermal grid consisting of  $N$  agents with heterogeneous parameters as it was developed in the previous part. Such a STG setting however can lead to unwanted mutual interactions between ATES systems as it is illustrated in Fig. 5. We therefore need to introduce a proper coupling constraint between neighboring agents, that makes use of the following assumption.

**Assumption 2.** Each well of an ATES system is considered as a growing reservoir with respect to the horizontal axis (Fig. 4, black solid line). We therefore assume to have a cylindrical reservoir with a fixed height  $\ell[m]$  (filter screen length) and a growing radius  $r_{a,t}^h, r_{a,t}^c[m]$  (thermal radius) for each well of an ATES system.

Using the volume of stored water in each well of ATES system, one can determine the thermal radius using:

$$r_{a,t}^h = \left( \frac{c_{pw} V_{a,t}^h}{c_{aq} \pi \ell} \right)^{0.5}, \quad r_{a,t}^c = \left( \frac{c_{pw} V_{a,t}^c}{c_{aq} \pi \ell} \right)^{0.5}, \quad (14)$$

where  $c_{aq} = (1 - n_p)c_{sand} + n_p c_{pw}$  is the aquifer heat capacity.  $c_{sand}$  [Jkg<sup>-1</sup>K<sup>-1</sup>] relates to the sand specific heat capacity, and  $n_p$  [-] is the porosity of aquifer. Let us now define the set of neighbors of agent  $i$  by

$$\mathcal{N}_i \subseteq \{1, 2, \dots, N\} \setminus \{i\}.$$

We impose a limit on the thermal radius of warm well  $r_{a,t}^h$  and cold well  $r_{a,t}^c$  of ATES system in each agent  $i$ , based on the corresponding wells of its neighbor  $j \in \mathcal{N}_i$ :

$$(r_{a,t}^h)_i + (r_{a,t}^c)_j \leq d_{ij}, \quad j \in \mathcal{N}_i, \quad (15)$$

where  $d_{ij}$  is a given distance between agent  $i$  and its neighbor  $j \in \mathcal{N}_i$ . This constraint prevents overlapping between the growing domains of warm and cold wells of ATES systems in a STG setting. Due to the nonlinear transformation in (14), we propose the following reformulation of this constraint to simplify the problem:

$$(V_{a,t}^h)_i + (V_{a,t}^c)_j \leq V_{ij} - \bar{\delta}_{ij,t}, \quad (16)$$

where  $V_{ij} = c_{aq} \pi \ell (d_{ij})^2 / c_{pw}$  denotes the total volume of common resource pool between agent  $i$  and its neighbor  $j \in \mathcal{N}_i$ .  $\bar{\delta}_{ij,t} = 2c_{aq} \pi \ell (\bar{r}_{a,t}^h)_i (\bar{r}_{a,t}^c)_j / c_{pw}$  represents a time-varying parameter that captures the mismatch between the linear and nonlinear constraint relations. The following corollary is a direct result of the above reformulation.

**Corollary 1.** *If  $(\bar{r}_{a,t}^h)_i$  and  $(\bar{r}_{a,t}^c)_j$  represent the current thermal radius of warm and cold wells of ATES system in agent  $i$  and  $j$ , respectively, then constraints (15) and (16) are equivalent.*

*Proof.* Substituting the corresponding relationships, we have

$$\begin{aligned} (V_{a,t}^h)_i + (V_{a,t}^c)_j &\leq V_{ij} - \bar{\delta}_{ij,t}, \\ \frac{c_{aq} \pi \ell}{c_{pw}} (\bar{r}_{a,t}^h)_i^2 + \frac{c_{aq} \pi \ell}{c_{pw}} (\bar{r}_{a,t}^c)_j^2 &\leq \frac{c_{aq} \pi \ell}{c_{pw}} (d_{ij})^2 - 2 \frac{c_{aq} \pi \ell}{c_{pw}} (\bar{r}_{a,t}^h)_i (\bar{r}_{a,t}^c)_j, \\ (\bar{r}_{a,t}^h)_i^2 + (\bar{r}_{a,t}^c)_j^2 &\leq (d_{ij})^2 - 2(\bar{r}_{a,t}^h)_i (\bar{r}_{a,t}^c)_j, \\ \left( (\bar{r}_{a,t}^h)_i + (\bar{r}_{a,t}^c)_j \right)^2 &\leq (d_{ij})^2, \\ (\bar{r}_{a,t}^h)_i + (\bar{r}_{a,t}^c)_j &\leq d_{ij}. \end{aligned}$$

The proof is completed by noting that the thermal radius is:  $(\bar{r}_{a,t}^h)_i \geq 0, \forall i \in \{1, \dots, N\}$ .  $\square$

**Definition 1.** *We define  $\delta_{ij,t}$  to be a common uncertainty source between each agent  $i$  and its neighboring agent  $j \in \mathcal{N}_i$ , using the following model:*

$$\delta_{ij,t} := \bar{\delta}_{ij,t} (1 \pm 0.1 \zeta), \quad (17)$$

where  $\zeta$  is a random variable defined on some probability space,  $\bar{\delta}_{ij,t}$  is constructed by using two given possible  $(\bar{r}_{a,t}^h)_i, (\bar{r}_{a,t}^c)_j$  realizations that can be obtained using historical data in the DS framework. Since the mapping (17) from  $\zeta$  to  $\delta_{ij,t}$  is measurable, one can view  $\delta_{ij,t}$  as a random variable on the same probability space as  $\zeta$ .

### 3.3. Problem Formulation in Multi-Agent Network

We now formulate the energy management problem for ATES systems in STGs as follows:

$$\min_{\{\mathbf{u}_i, \mathbf{v}_i\}_{i=1}^N} \sum_{i=1}^N \mathcal{V}_i(\mathbf{x}_i, \mathbf{u}_i) \quad (18a)$$

$$\text{s.t.} \quad E_i \mathbf{u}_i + F_i \mathbf{v}_i + P_i \leq 0, \quad (18b)$$

$$\mathbb{P}_{\mathbf{w}_i} \left[ A_i \mathbf{x}_{i,k} + B_i \mathbf{u}_i + C_i \mathbf{w}_i \geq 0 \right] \geq 1 - \varepsilon_i, \quad \forall \mathbf{w}_i \in \mathcal{W}_i, \quad (18c)$$

$$\mathbb{P}_{\boldsymbol{\delta}_{ij}} \left[ H_i \mathbf{x}_i + H_j \mathbf{x}_j \leq \bar{V}_{ij} - \boldsymbol{\delta}_{ij} \right] \geq 1 - \bar{\varepsilon}_{ij}, \quad \forall \boldsymbol{\delta}_{ij} \in \Delta_{ij}, \quad \forall j \in \mathcal{N}_i, \quad (18d)$$

$$\forall i \in \{1, 2, \dots, N\},$$

where  $H_i, H_j$  are matrices of appropriate dimensions,  $\bar{V}_{ij} \in \mathbb{R}^{N_h}$  is the upper-bound on the total common resource pool,  $\boldsymbol{\delta}_{ij}$  is a vector of common uncertainty variables, and  $\bar{\varepsilon}_{ij} \in (0, 1)$  denotes the level of admissible coupling constraint violation, for each agent  $i$  and  $\forall j \in \mathcal{N}_i$ .  $\bar{V}_{ij}$  can be expressed as  $\bar{V}_{ij} = \mathbf{1}^{N_h} \otimes V_{ij}$ , using the Kronecker product. It is important to notice that the index of  $\mathbb{P}_{\boldsymbol{\delta}_{ij}}$  denotes the dependency of the state trajectories on the string of random common scenarios  $\boldsymbol{\delta}_{ij} = [\delta_{ij,k}, \delta_{ij,k+1}, \dots, \delta_{ij,k+N_h}] \subseteq \mathbb{R}^{N_h=n_\delta}$ .

**Assumption 3.** Following Definition 1,  $\boldsymbol{\delta}_{ij}$  is defined on some probability space  $(\Delta_{ij}, \mathfrak{B}(\Delta_{ij}), \mathbb{P}_{\boldsymbol{\delta}_{ij}})$ , where  $\Delta_{ij} \subseteq \mathbb{R}^{n_\delta}$ ,  $\mathfrak{B}(\cdot)$  denotes a Borel  $\sigma$ -algebra, and  $\mathbb{P}_{\boldsymbol{\delta}_{ij}}$  is a probability measure defined over  $\Delta_{ij}$ .

**Assumption 4.**  $\mathbf{w}_i \in \mathbb{R}^{n_w}$  and  $\boldsymbol{\delta}_{ij} \in \mathbb{R}^{n_\delta}$  are two independent string of random scenarios from two disjoint probability space  $\mathcal{W}_i$  and  $\Delta_{ij}$ , respectively.

We refer to the proposed optimization problem (18) as a multi-agent network problem, and whenever the proposed problem (18) is solved in a receding horizon fashion, it is mentioned as the *centralized solution* (CS) in the following parts. However, the feasible set in (18) is in general non-convex and hard to determine explicitly due to the presence of chance constraints (18c), (18d). In what follows, we develop a tractable framework to obtain probabilistically feasible solutions for all agents.

### 3.4. Move-Blocking Scheme

The proposed system dynamics (9) for each agent  $i = 1, 2, \dots, N$ , represents a system where the building climate comfort systems (2) have an hourly-based operation and typically day-ahead planning compared to the ATES system (3) that is based on a seasonal variation of desired optimal operation. This leads to a problem formulation (18) that is sensitive w.r.t. the prediction horizon length. Using a fixed prediction horizon length, e.g. least common multiple of these two systems, may turn out to be computationally prohibitive however also necessary in order to represent ATES interaction dynamics. We therefore aim to formulate a move-blocking strategy to reduce the number of control variables.

Consider  $\mathcal{N}_h = \{k, k+1, \dots, k+N_h-1\}$  to be the set of sampling time instances through full prediction horizon, and  $\mathcal{T} = \{\tau_1, \tau_2, \dots, \tau_T\} \subseteq \mathcal{N}_h$  to be the set of sampling time instances at which the control input is updated. We introduce a new vector of multi-rate decision variables  $\tilde{\mathbf{u}}_i \in \mathbb{R}^{N_u T}$  which are related to the original ones by:

$$\mathbf{u}_i = \boldsymbol{\Psi} \tilde{\mathbf{u}}_i, \quad (19)$$

where  $\boldsymbol{\Psi} = [\Psi_1 \quad \Psi_2 \quad \dots \quad \Psi_T] \in \mathbb{R}^{N_u N_h \times N_u T}$  is a linear mapping matrix. For all  $m \in \{1, \dots, T\}$ , we construct

$$\Psi_m = [\psi_{1,m}^\top \quad \psi_{2,m}^\top \quad \dots \quad \psi_{N_h,m}^\top]^\top \in \mathbb{R}^{N_u N_h \times N_u}, \quad (20)$$

where  $\psi_{l,m} \in \mathbb{R}^{N_u \times N_u}$  for all  $l \in \{1, 2, \dots, N_h\}$  is defined as

$$\psi_{l,m} = \begin{cases} \mathbf{1} & \text{if } k + l - 1 = \tau_m, \\ 0 & \text{otherwise} \end{cases}, \quad (21)$$

where  $\mathbf{1} \in \mathbb{R}^{N_u \times N_u}$  represents an identity matrix.

We reformulate the optimization problem (18) using the proposed move-blocking scheme (19), and whenever the reformulation of (18) is solved in a receding horizon fashion, it is referred to as the *move-blocking centralized solution* (MCS).

#### 4. COMPUTATIONALLY TRACTABLE FRAMEWORK

In this section, we develop a tractable methodology to reformulate the proposed chance-constrained optimization problem (18), which is in general difficult to solve. Using data driven approach, the randomization (scenario-based) technique, one can simply extract at random some instances of the uncertainty (scenarios), where the scenarios are independent and identically distributed (i.i.d.), and 2) find the optimal solution of problem with only the constraints associated with the extracted scenarios. An important requirement of such a technique is to have a convex problem w.r.t the decision variables, which is not the case in our problem formulation (18).

To tackle such a mixed-integer chance-constrained problem, one can use a worst-case mixed-integer reformulation technique as it was initially introduced in [17]. Due to the large-scale network problem (18), such a reformulation leads to enormous cost of computation time and it is indeed an intractable approach. Following the so-called robust randomized technique, the reformulation is done in a way to provide a feasible solution for all scenarios of the uncertainty realizations in a probabilistic sense. The idea of this approach is straightforward. An auxiliary chance-constrained optimization problem is first formulated to determine a probabilistic bounded set of uncertainty realizations. This yields a bounded set of uncertainty that is a subset of the uncertainty space and contains a portion of the probability mass of the uncertainty with high confidence level. Then a robust version of the initial problem subject to the uncertainty confined in the obtained set is solved. We here extend this framework such that to be able to handle a problem with multiple chance constraints based on the idea of robust randomized approach.

Consider  $\mathbf{y}_i = (\mathbf{u}_i, \mathbf{v}_i) \in \mathbb{R}^{(n_u+n_v)=n_y}$ ,  $\mathbf{y} = \text{col}(\mathbf{y}_i)_{i=1}^N$ , where  $\text{col}(\cdot)$  is an operator to stack elements. Define  $\mathbf{w} = \text{col}(\mathbf{w}_i)_{i=1}^N \subseteq \mathcal{W}$  to be the private uncertainty sources for a network of agents,  $\boldsymbol{\delta}_i = \text{col}(\boldsymbol{\delta}_j)_{j \in \mathcal{N}_i} \subseteq \Delta_i$  to be the common uncertainty sources for each agent, and  $\boldsymbol{\delta} = \text{col}(\boldsymbol{\delta}_i)_{i=1}^N \subseteq \Delta$  to be the common uncertainty sources for a multi-agent network, where

$$\mathcal{W} := \prod_{i=1}^N \mathcal{W}_i, \quad \Delta_i := \prod_{j \in \mathcal{N}_i} \Delta_{ij}, \quad \Delta := \prod_{i=1}^N \Delta_i.$$

Consider now the proposed optimization problem in (18) in a more compact format:

$$\underset{\mathbf{y}}{\text{minimize}} \quad \sum_{i=1}^N \mathcal{V}_i(\mathbf{x}_i, \mathbf{u}_i) \quad (22a)$$

$$\text{subject to} \quad \mathbb{P}_{\mathbf{w}} \left[ \mathbf{y} \in \prod_{i=1}^N \mathcal{Y}_i(\mathbf{w}_i) \right] \geq 1 - \varepsilon, \quad \forall \mathbf{w} \in \mathcal{W} \quad (22b)$$

$$\mathbb{P}_{\boldsymbol{\delta}} \left[ \mathbf{y} \in \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) \right] \geq 1 - \bar{\varepsilon}, \quad \forall \boldsymbol{\delta} \in \Delta \quad (22c)$$

where  $\varepsilon := \sum_{i=1}^N \varepsilon_i \in (0, 1)$ ,  $\bar{\varepsilon} := \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \bar{\varepsilon}_{ij} \in (0, 1)$ .  $\mathcal{Y}_i(\mathbf{w}_i) \in \mathbb{R}^{n_y}$  and  $\mathcal{Y}_{ij}(\boldsymbol{\delta}_{ij}) \in \mathbb{R}^{n_y}$  are defined<sup>1</sup> by

$$\begin{aligned} \mathcal{Y}_i(\mathbf{w}_i) &:= \left\{ \mathbf{y}_i \in \mathbb{R}^{n_y} : E_i \mathbf{u}_i + F_i \mathbf{v}_i + P_i \leq 0, A_i x_{i,k} + B_i \mathbf{u}_i + C_i \mathbf{w}_i \geq 0 \right\}, \\ \mathcal{Y}_{ij}(\boldsymbol{\delta}_{ij}) &:= \left\{ (\mathbf{y}_i, \mathbf{y}_j) \in \mathbb{R}^{2n_y} : H_i \mathbf{x}_i + H_j \mathbf{x}_j \leq \bar{V}_{ij} - \boldsymbol{\delta}_{ij} \right\}. \end{aligned}$$

It is important to notice that  $\check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) \in \mathbb{R}^{2n_y N_i}$  represents the cylindrical extension<sup>2</sup> of  $\mathcal{Y}_{ij}(\boldsymbol{\delta}_{ij})$ . In the subsequent parts, we refer to the constraint (22b) as the agents' private chance constraints, and to the constraint (22c) as the agents' common chance constraints.

The proposed formulation (22) is a mixed-integer quadratic optimization problem with multiple chance constraints, due to the binary variables  $\{\mathbf{v}_i\}_{i=1}^N$ , and the chance constraints (22b), (22c). It is worth to mention that the index of  $\mathbb{P}_{\mathbf{w}}$  and  $\mathbb{P}_{\boldsymbol{\delta}}$  denote the dependency on the string of random scenarios  $\mathbf{w} \in \mathcal{W}$  and  $\boldsymbol{\delta} \in \Delta$ , respectively.

Building upon our previous work in [8], we extend the so-called robust randomized approach in [11] to be more applicable to handle a problem with multiple chance constraints. The proposed optimization problem (22) is a stochastic program with multiple chance constraints, where  $\mathbb{P}_{\mathbf{w}}$  and  $\mathbb{P}_{\boldsymbol{\delta}}$  denote two different probability measures for private uncertainty and common uncertainty sources, respectively. In summary, the formulation in [18, Proposition 1] considered a worst-case chance constraint defined by

$$\max_{k \in \mathcal{N}_{\text{MCP}}} \mathbb{P}[f_k(\mathbf{y}, \cdot)] \geq 1 - \tilde{\varepsilon}, \quad (23)$$

where  $\tilde{\varepsilon} = \min_{k \in \mathcal{N}_{\text{MCP}}} \{\varepsilon_k\}$ ,  $f_k(\mathbf{y}, \cdot)$  denotes the  $k$ -th chance constraint function, and  $\mathcal{N}_{\text{MCP}}$  is the set of indices of chance constraint functions formulated in the proposed optimization problem (22). However, this procedure leads to a considerable amount of conservatism, due to the fact that it requires the solution to satisfy all constraints with the highest probability  $1 - \tilde{\varepsilon}$ . We instead employ the robust randomized approach for each chance constraint function  $f_k(\mathbf{y}, \cdot)$ ,  $k \in \mathcal{N}_{\text{MCP}}$ , separately. Our framework is closely related to, albeit different from the approach of [12], since the feasible set in (22) is non-convex. Moreover, the problem formulation in [12] consists of an objective function with multiple chance constraints, in which the terms in objective and constraints are univariate w.r.t. the decision variables. In contrast the objective function in our problem formulation (22) consists of separable additive components and constraint functions are also separable w.r.t. (22b), (22c) between each agent  $i = 1, \dots, N$  and  $\forall j \in \mathcal{N}_i$ .

We define  $\mathcal{B}_i$ ,  $\bar{\mathcal{B}}_{ij}$  to be two bounded sets of private uncertainty source and a bounded set of common uncertainty source for each agent  $i$ , respectively.  $\mathcal{B}_i$ ,  $\bar{\mathcal{B}}_{ij}$  are assumed to be axis-aligned hyper-rectangular sets. This is not restrictive and any convex set with convex volume could have been chosen instead as in [19]. We parametrize  $\mathcal{B}_i(\boldsymbol{\gamma}) := [\bar{\boldsymbol{\gamma}}, \underline{\boldsymbol{\gamma}}]$  by  $\boldsymbol{\gamma} = (\bar{\boldsymbol{\gamma}}, \underline{\boldsymbol{\gamma}}) \in \mathbb{R}^{2n_w}$ , and  $\bar{\mathcal{B}}_{ij}(\boldsymbol{\lambda}) := [\bar{\boldsymbol{\lambda}}, \underline{\boldsymbol{\lambda}}]$  by  $\boldsymbol{\lambda} = (\bar{\boldsymbol{\lambda}}, \underline{\boldsymbol{\lambda}}) \in \mathbb{R}^{2n_\delta}$ , and consider the following chance-constrained optimization problem:

$$\begin{cases} \min_{\boldsymbol{\gamma}} & \|\bar{\boldsymbol{\gamma}} - \underline{\boldsymbol{\gamma}}\|_1 \\ \text{s.t.} & \mathbb{P}\{\mathbf{w}_i \in \mathcal{W}_i \mid \mathbf{w}_i \in [\underline{\boldsymbol{\gamma}}, \bar{\boldsymbol{\gamma}}]\} \geq 1 - \varepsilon_i \end{cases}, \quad (24a)$$

$$\begin{cases} \min_{\boldsymbol{\lambda}} & \|\bar{\boldsymbol{\lambda}} - \underline{\boldsymbol{\lambda}}\|_1 \\ \text{s.t.} & \mathbb{P}\{\boldsymbol{\delta}_{ij} \in \Delta_{ij} \mid \boldsymbol{\delta}_{ij} \in [\underline{\boldsymbol{\lambda}}, \bar{\boldsymbol{\lambda}}]\} \geq 1 - \bar{\varepsilon}_{ij} \end{cases}. \quad (24b)$$

Following the so-called scenario approach in [20], one can determine the number of required uncertainty scenarios to formulate a tractable problem, using  $N_s = \frac{2}{\epsilon}(\xi + \ln \frac{1}{\nu})$ , where  $\xi$  is the dimension of decision vector,

<sup>1</sup>Both sets have a dependency on the initial value of the state  $x_{i,k}$  for each agent  $i$  at each sampling time  $k$ . Given  $x_{i,k}$ , we here highlight the dependency of these sets on the uncertainties  $\mathbf{w}_i$  and  $\boldsymbol{\delta}_{ij}$  for each agent  $i$  at each sampling time  $k$ .

<sup>2</sup>Cylindrical extension simply replicates the membership degrees from the existing dimensions into the new dimensions.



$\epsilon, \nu$  are the level of violation, and the confidence level, respectively. We once substitute  $\xi = 2n_w, \epsilon = \epsilon_i, \nu = \beta_i$ , determine  $N_{s_i}$ , and once  $\xi = 2n_\delta, \epsilon = \bar{\epsilon}_{ij}, \nu = \bar{\beta}_{ij}$ , determine  $\bar{N}_{s_{ij}}$ , for all agent  $i \in \{1, 2, \dots, N\}$ . We next define  $\mathcal{S}_i = \{\mathbf{w}_i^{(1)}, \dots, \mathbf{w}_i^{(N_{s_i})}\} \subset \mathcal{W}_i, \bar{\mathcal{S}}_{ij} = \{\boldsymbol{\delta}_{ij}^{(1)}, \dots, \boldsymbol{\delta}_{ij}^{(\bar{N}_{s_{ij}})}\} \subset \Delta_{ij}$  and formulate a tractable version of (24a) and (24b) by

$$\begin{cases} \min_{\boldsymbol{\gamma}} & \|\bar{\boldsymbol{\gamma}} - \underline{\boldsymbol{\gamma}}\|_1 \\ \text{s.t.} & \mathbf{w}_i \in [\underline{\boldsymbol{\gamma}}, \bar{\boldsymbol{\gamma}}] \quad , \quad \mathbf{w}_i \in \mathcal{S}_i \end{cases} , \quad (25a)$$

$$\begin{cases} \min_{\boldsymbol{\lambda}} & \|\bar{\boldsymbol{\lambda}} - \underline{\boldsymbol{\lambda}}\|_1 \\ \text{s.t.} & \boldsymbol{\delta}_{ij} \in [\underline{\boldsymbol{\lambda}}, \bar{\boldsymbol{\lambda}}] \quad , \quad \boldsymbol{\delta}_{ij} \in \bar{\mathcal{S}}_{ij} \end{cases} . \quad (25b)$$

The optimal solutions  $(\boldsymbol{\gamma}^*, \boldsymbol{\lambda}^*)$  of the proposed tractable problem are probabilistically feasible for the chance-constrained problems, [21, Theorem 1]. Moreover,  $\boldsymbol{\gamma}^*$ , and  $\boldsymbol{\lambda}^*$  also characterize our desired probabilistic bounded sets  $\mathcal{B}_i^*$  and  $\bar{\mathcal{B}}_{ij}^*$ , respectively.

**Assumption 5.**  $\mathcal{S}_i$  and  $\bar{\mathcal{S}}_{ij}$  are two collections of random scenarios that are i.i.d.

After determining  $\mathcal{B}_i^*$  and  $\bar{\mathcal{B}}_{ij}^*$  for all agents  $i \in \{1, \dots, N\}$ , we are now able to reformulate the robust counterpart of the original problem (22) via:

$$\min_{\mathbf{y}} \quad \sum_{i=1}^N \mathcal{V}_i(\mathbf{x}_i, \mathbf{u}_i) \quad (26a)$$

$$\text{s.t.} \quad \mathbf{y} \in \prod_{i=1}^N \bigcap_{\mathbf{w}_i \in \{\mathcal{B}_i^* \cap \mathcal{W}_i\}} \mathcal{Y}_i(\mathbf{w}_i) , \quad (26b)$$

$$\mathbf{y} \in \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \bigcap_{\boldsymbol{\delta}_{ij} \in \{\bar{\mathcal{B}}_{ij}^* \cap \Delta_{ij}\}} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) . \quad (26c)$$

Note that the aforementioned problem is not a randomized program, and instead, the constraints have to be satisfied for all values of the private uncertainty in  $\{\mathcal{B}_i^* \cap \mathcal{W}_i\}$ , and common uncertainty in  $\{\bar{\mathcal{B}}_{ij}^* \cap \Delta_{ij}\}$ . The proposed problem (26) is a robust mixed-integer quadratic program. In [22], it was shown that the robust problems are tractable and remain in the same class as the original problems, e.g. robust mixed-integer programs remain mixed-integer programs, for a certain class of uncertainty sets, such as in our problem (26), the uncertainty is bounded in a convex set. The following theorem quantifies the robustness of solution obtained by (26) w.r.t. the initial problem (22).

**Theorem 1.** Let  $\epsilon_i, \bar{\epsilon}_i, \bar{\epsilon}_{ij}, \epsilon, \bar{\epsilon}, \beta_i, \bar{\beta}_i, \bar{\beta}_{ij}, \beta, \bar{\beta} \in (0, 1), \forall j \in \mathcal{N}_i, i = 1, 2, \dots, N$  be chosen such that  $\epsilon = \sum_{i=1}^N \epsilon_i, \beta = \sum_{i=1}^N \beta_i, \bar{\epsilon}_i = \sum_{j \in \mathcal{N}_i} \bar{\epsilon}_{ij}, \bar{\beta}_i = \sum_{j \in \mathcal{N}_i} \bar{\beta}_{ij}$ , and  $\bar{\epsilon} = \sum_{i=1}^N \bar{\epsilon}_i, \bar{\beta} = \sum_{i=1}^N \bar{\beta}_i$ . If  $\mathbf{y}_s^*$  is a feasible solution of the problem (26), then  $\mathbf{y}_s^*$  is also a feasible solution for the chance constraints (22b) and (22c), with the confidence levels of  $1 - \beta$  and  $1 - \bar{\beta}$ , respectively.

*Proof.* Due to the fact that the proposed optimization problem (26) has multiple chance constraints, we modify the results in [18, Proposition 1] to be able to handle this new situation. In particular, the proof follows a similar steps as in [18, Proposition 1] for each constraint, separately. To this purpose, we define  $\text{Vio}^p(\mathbf{y}_s^*)$ , and  $\text{Vio}^c(\mathbf{y}_s^*)$  to be the violation probability of the private and common chance constraints as in (22b), and (22c), respectively. They can be quantified by

$$\text{Vio}^p(\mathbf{y}_s^*) = \mathbb{P}_{\mathbf{w}} \left[ \mathbf{w} \in \mathcal{W} : \mathbf{y}_s^* \notin \prod_{i=1}^N \mathcal{Y}_i(\mathbf{w}_i) , \mathbf{y}_s^* \in \mathcal{Y} \right] , \quad (27a)$$



$$\text{Vio}^c(\mathbf{y}_s^*) = \mathbb{P}_\delta \left[ \boldsymbol{\delta} \in \Delta : \mathbf{y}_s^* \notin \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) , \mathbf{y}_s^* \in \mathcal{Y} \right] , \quad (27b)$$

where  $\mathcal{Y}$  is the feasible region of (26), and it can be characterized via

$$\mathcal{Y} := \left\{ \mathbf{y} \in \mathbb{R}^{n_y N} : \mathbf{y} \in \left\{ \prod_{i=1}^N \bigcap_{\mathbf{w}_i \in \{\mathcal{B}_i^* \cap \mathcal{W}_i\}} \mathcal{Y}_i(\mathbf{w}_i) \right\} \cap \left\{ \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \bigcap_{\boldsymbol{\delta}_{ij} \in \{\bar{\mathcal{B}}_{ij}^* \cap \Delta_{ij}\}} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) \right\} \right\} .$$

It is important to mention that both definitions in (27) are conditional probabilities  $\mathbb{P}_{\mathbf{w}}, [(\cdot) | \forall \boldsymbol{\delta} \in \Delta]$ , and  $\mathbb{P}_\delta [(\cdot) | \forall \mathbf{w} \in \mathcal{W}]$ . Following 4, we equivalently considered them in the form of eq. (27a) and eq. (27b), almost surely. Define  $\mathcal{B}^* = \prod_{i=1}^N \mathcal{B}_i^*$ ,  $\bar{\mathcal{B}}^* = \prod_{i=1}^N \bar{\mathcal{B}}_i^*$ , and  $\bar{\mathcal{B}}_i^* = \prod_{j \in \mathcal{N}_i} \bar{\mathcal{B}}_{ij}^*$ , and clearly, we can have:

$$\begin{aligned} \mathbf{y}^* \in \prod_{i=1}^N \bigcap_{\mathbf{w}_i \in \{\mathcal{B}_i^* \cap \mathcal{W}_i\}} \mathcal{Y}_i(\mathbf{w}_i) &\leftrightarrow \mathbf{y}^* \in \bigcap_{\mathbf{w} \in \{\mathcal{B}^* \cap \mathcal{W}\}} \prod_{i=1}^N \mathcal{Y}_i(\mathbf{w}_i) \\ \mathbf{y}^* \in \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \bigcap_{\boldsymbol{\delta}_{ij} \in \{\bar{\mathcal{B}}_{ij}^* \cap \Delta_{ij}\}} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) &\leftrightarrow \mathbf{y}^* \in \prod_{i=1}^N \bigcap_{\boldsymbol{\delta}_i \in \{\bar{\mathcal{B}}_i^* \cap \Delta_i\}} \bigcap_{j \in \mathcal{N}_i} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) \\ \mathbf{y}^* \in \prod_{i=1}^N \bigcap_{\boldsymbol{\delta}_i \in \{\bar{\mathcal{B}}_i^* \cap \Delta_i\}} \bigcap_{j \in \mathcal{N}_i} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) &\leftrightarrow \mathbf{y}^* \in \bigcap_{\boldsymbol{\delta} \in \{\bar{\mathcal{B}}^* \cap \Delta\}} \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) \end{aligned}$$

Therefore, if  $\mathbf{w} \in \{\mathcal{B}^* \cap \mathcal{W}\}$  then  $\mathbf{y}^* \in \prod_{i=1}^N \mathcal{Y}_i(\mathbf{w}_i)$ , and if  $\boldsymbol{\delta} \in \{\bar{\mathcal{B}}^* \cap \Delta\}$  then  $\mathbf{y}^* \in \prod_{i=1}^N \bigcap_{j \in \mathcal{N}_i} \check{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij})$ . This yields the following relations:

$$\text{Vio}^p(\mathbf{y}_s^*) \leq \mathbb{P}_{\mathbf{w}} [\mathbf{w} \in \mathcal{W} : \mathbf{w} \notin \mathcal{B}^*] = \text{Vio}(\mathcal{B}^*) , \quad (28a)$$

$$\text{Vio}^c(\mathbf{y}_s^*) \leq \mathbb{P}_\delta [\boldsymbol{\delta} \in \Delta : \boldsymbol{\delta} \notin \bar{\mathcal{B}}^*] = \text{Vio}(\bar{\mathcal{B}}^*) , \quad (28b)$$

It is then sufficient to show that for  $N_s = \max_{i=1, \dots, N} N_{s_i}$ , and  $\bar{N}_s = \max_{i=1, \dots, N} \max_{j \in \mathcal{N}_i} \bar{N}_{s_{ij}}$ :

$$\mathbb{P}_{\mathbf{w}}^{N_s} [\mathcal{S} \in \mathcal{W}^{N_s} : \text{Vio}(\mathcal{B}^*) \geq \varepsilon] \leq \beta , \quad (29a)$$

$$\mathbb{P}_\delta^{\bar{N}_s} [\bar{\mathcal{S}} \in \Delta^{\bar{N}_s} : \text{Vio}(\bar{\mathcal{B}}^*) \geq \bar{\varepsilon}] \leq \bar{\beta} , \quad (29b)$$

where  $\mathcal{S} = \prod_{i=1}^N \mathcal{S}_i$ , and  $\bar{\mathcal{S}} = \prod_{i=1}^N \prod_{j \in \mathcal{N}_i} \bar{\mathcal{S}}_{ij}$ . To this end, we now break down the proof in the following steps to show (28) and (29):

a) Private chance constraint violation:

$$\begin{aligned} \text{Vio}^p(\mathbf{y}_s^*) &\leq \text{Vio}(\mathcal{B}^*) = \mathbb{P}_{\mathbf{w}} [\mathbf{w} \in \mathcal{W} : \mathbf{w} \notin \mathcal{B}^*] \\ &= \mathbb{P}_{\mathbf{w}} \left[ \mathbf{w} \in \mathcal{W} : \mathbf{w} \notin \prod_{i=1}^N \mathcal{B}_i^* \right] \\ &= \mathbb{P}_{\mathbf{w}} [\mathbf{w} \in \mathcal{W} : \exists i \in \{1, \dots, N\}, \mathbf{w}_i \notin \mathcal{B}_i^*] \\ &= \mathbb{P}_{\mathbf{w}} \left[ \bigcup_{i=1}^N \{\mathbf{w}_i \in \mathcal{W}_i : \mathbf{w}_i \notin \mathcal{B}_i^*\} \right] \\ &\leq \sum_{i=1}^N \mathbb{P}_{\mathbf{w}_i} [\mathbf{w}_i \in \mathcal{W}_i : \mathbf{w}_i \notin \mathcal{B}_i^*] = \sum_{i=1}^N \text{Vio}(\mathcal{B}_i^*) . \end{aligned}$$

The last statement implies that  $\text{Vio}^p(\mathbf{y}_s^*) \leq \sum_{i=1}^N \text{Vio}(\mathcal{B}_i^*)$ , and thus, we have

$$\begin{aligned} \mathbb{P}_{\mathbf{w}}^{N_s} [\mathcal{S} \in \mathcal{W}^{N_s} : \text{Vio}^p(\mathbf{y}_s^*) \geq \varepsilon] &\leq \mathbb{P}_{\mathbf{w}}^{N_s} \left[ \mathcal{S} \in \mathcal{W}^{N_s} : \sum_{i=1}^N \text{Vio}(\mathcal{B}_i^*) \geq \sum_{i=1}^N \varepsilon_i \right] \\ &= \mathbb{P}_{\mathbf{w}}^{N_s} \left[ \bigcup_{i=1}^N \left\{ \mathcal{S}_i \in \mathcal{W}_i^{N_{s_i}} : \text{Vio}(\mathcal{B}_i^*) \geq \varepsilon_i \right\} \right] \\ &\leq \sum_{i=1}^N \mathbb{P}_{\mathbf{w}_i}^{N_{s_i}} \left[ \mathcal{S}_i \in \mathcal{W}_i^{N_{s_i}} : \text{Vio}(\mathcal{B}_i^*) \geq \varepsilon_i \right] \leq \sum_{i=1}^N \beta_i = \beta. \end{aligned}$$

b) Common chance constraint violation:

$$\begin{aligned} \text{Vio}^c(\mathbf{y}_s^*) &\leq \text{Vio}(\bar{\mathcal{B}}^*) = \mathbb{P}_{\boldsymbol{\delta}} [\boldsymbol{\delta} \in \Delta : \boldsymbol{\delta} \notin \bar{\mathcal{B}}^*] \\ &= \mathbb{P}_{\boldsymbol{\delta}} \left[ \boldsymbol{\delta} \in \Delta : \boldsymbol{\delta} \notin \prod_{i=1}^N \prod_{j \in \mathcal{N}_i} \bar{\mathcal{B}}_{ij}^* \right] \\ &= \mathbb{P}_{\boldsymbol{\delta}} \left[ \boldsymbol{\delta} \in \Delta : \exists i \in \{1, \dots, N\}, \boldsymbol{\delta}_i \notin \prod_{j \in \mathcal{N}_i} \bar{\mathcal{B}}_{ij}^* \right] \\ &= \mathbb{P}_{\boldsymbol{\delta}} \left[ \bigcup_{i=1}^N \left\{ \boldsymbol{\delta}_i \in \Delta_i : \boldsymbol{\delta}_i \notin \prod_{j \in \mathcal{N}_i} \bar{\mathcal{B}}_{ij}^* \right\} \right] \\ &\leq \sum_{i=1}^N \mathbb{P}_{\boldsymbol{\delta}_i} [\boldsymbol{\delta}_i \in \Delta_i : \exists j \in \mathcal{N}_i, \boldsymbol{\delta}_{ij} \notin \bar{\mathcal{B}}_{ij}^*] \\ &= \sum_{i=1}^N \mathbb{P}_{\boldsymbol{\delta}_i} \left[ \bigcup_{j \in \mathcal{N}_i} \left\{ \boldsymbol{\delta}_{ij} \in \Delta_{ij} : \boldsymbol{\delta}_{ij} \notin \bar{\mathcal{B}}_{ij}^* \right\} \right] \\ &\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{P}_{\boldsymbol{\delta}_{ij}} [\boldsymbol{\delta}_{ij} \in \Delta_{ij} : \boldsymbol{\delta}_{ij} \notin \bar{\mathcal{B}}_{ij}^*] = \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \text{Vio}(\bar{\mathcal{B}}_{ij}^*). \end{aligned}$$

This implies that  $\text{Vio}(\mathbf{y}_s^*) \leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \text{Vio}(\bar{\mathcal{B}}_{ij}^*)$ , and thus, we have

$$\begin{aligned} \mathbb{P}_{\boldsymbol{\delta}}^{\bar{N}_s} [\bar{\mathcal{S}} \in \Delta^{\bar{N}_s} : \text{Vio}(\mathbf{y}_s^*) \geq \bar{\varepsilon}] &\leq \mathbb{P}_{\boldsymbol{\delta}}^{\bar{N}_s} \left[ \bar{\mathcal{S}} \in \Delta^{\bar{N}_s} : \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \text{Vio}(\bar{\mathcal{B}}_{ij}^*) \geq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \bar{\varepsilon}_{ij} \right] \\ &= \mathbb{P}_{\boldsymbol{\delta}}^{\bar{N}_s} \left[ \bigcup_{i=1}^N \left\{ \bar{\mathcal{S}}_i \in \Delta_i^{\bar{N}_{s_i}} : \sum_{j \in \mathcal{N}_i} \text{Vio}(\bar{\mathcal{B}}_{ij}^*) \geq \sum_{j \in \mathcal{N}_i} \bar{\varepsilon}_{ij} \right\} \right] \\ &\leq \sum_{i=1}^N \mathbb{P}_{\boldsymbol{\delta}_i}^{\bar{N}_{s_i}} \left[ \bar{\mathcal{S}}_i \in \Delta_i^{\bar{N}_{s_i}} : \sum_{j \in \mathcal{N}_i} \text{Vio}(\bar{\mathcal{B}}_{ij}^*) \geq \sum_{j \in \mathcal{N}_i} \bar{\varepsilon}_{ij} \right] \\ &= \sum_{i=1}^N \mathbb{P}_{\boldsymbol{\delta}_i}^{\bar{N}_{s_i}} \left[ \bigcup_{j \in \mathcal{N}_i} \left\{ \bar{\mathcal{S}}_{ij} \in \Delta_{ij}^{\bar{N}_{s_{ij}}} : \text{Vio}(\bar{\mathcal{B}}_{ij}^*) \geq \bar{\varepsilon}_{ij} \right\} \right] \\ &\leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \mathbb{P}_{\boldsymbol{\delta}_{ij}}^{\bar{N}_{s_{ij}}} \left[ \bar{\mathcal{S}}_{ij} \in \Delta_{ij}^{\bar{N}_{s_{ij}}} : \text{Vio}(\bar{\mathcal{B}}_{ij}^*) \geq \bar{\varepsilon}_{ij} \right] \leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \beta_{ij} = \beta. \end{aligned}$$

The obtained bounds in the above procedure are the desired assertions as it is stated in the theorem. It is important to mention that we use the existing results in [21] to determine  $N_{s_i}$  and  $\bar{N}_{s_{ij}}$  and solve the tractable problems (25a) and (25b) for each agent  $i = 1, \dots, N$ ,  $\forall j \in \mathcal{N}_i$ , respectively. We thus have the following probabilistic guarantees:

$$\mathbb{P}_{\mathbf{w}_i}^{N_{s_i}} \left[ \mathcal{S}_i \in \mathcal{W}_i^{N_{s_i}} : \text{Vio}(\mathcal{B}_i^*) \geq \varepsilon_i \right] \leq \beta_i, \quad \mathbb{P}_{\delta_{ij}}^{\bar{N}_{s_{ij}}} \left[ \bar{\mathcal{S}}_{ij} \in \Delta_{ij}^{\bar{N}_{s_{ij}}} : \text{Vio}(\bar{\mathcal{B}}_{ij}^*) \geq \bar{\varepsilon}_{ij} \right] \leq \beta_{ij}. \quad (30)$$

The interpretation of the derivation of these bounds (29) is as follows. The probability of all violation probabilities  $\text{Vio}(\mathcal{B}_i^*)$  being simultaneously bounded by the corresponding  $\varepsilon_i$  is at least  $1 - \beta$ , and  $\text{Vio}(\bar{\mathcal{B}}_{ij}^*)$  being simultaneously bounded by the corresponding  $\bar{\varepsilon}_{ij}$  is at least  $1 - \bar{\beta}$ . The proof is completed by noting that the feasible set  $\mathcal{Y}$  of (26) has a non-empty interior:

$$\left\{ \exists \rho \in \mathbb{R}_+, \bar{\mathbf{y}} \in \mathcal{Y} : \|\mathbf{y} - \bar{\mathbf{y}}\| \leq \rho, \forall \mathbf{y} \in \mathbb{R}^{n_y} \right\} \subset \mathcal{Y},$$

and since the problem (26) has a non-empty interior feasible set, it admits at least one feasible solution  $\mathbf{y}_s^*$ .  $\square$

**Remark 6.** Following the approach in [23], we approximate the objective function empirically for each agent  $i$ .  $\mathbb{E}_{\mathbf{w}_i}[\mathcal{J}_i(\cdot)]$  can be approximated by averaging the value of its argument for some number of different scenarios, which plays a tuning parameter role. Using  $N_{s_i^0}$  as the tuning parameter, consider  $N_{s_i^0}$  number of different scenarios of  $\mathbf{w}_i$  to build  $\mathcal{S}_i^0 = \{\mathbf{w}_i^{(1)}, \dots, \mathbf{w}_i^{(N_{s_i^0})}\} \subset \mathcal{W}_i$  for each agent  $i = 1, \dots, N$ . We then approximate the cost function empirically as follows:

$$\sum_{i=1}^N \mathcal{V}_i(\mathbf{x}_i(\mathbf{w}_i), \mathbf{u}_i) = \sum_{i=1}^N \mathbb{E}_{\mathbf{w}_i \in \mathcal{W}_i} [\mathcal{J}_i(\mathbf{x}_i(\mathbf{w}_i), \mathbf{u}_i)] \approx \sum_{i=1}^N \sum_{\mathbf{w}_i \in \mathcal{S}_i^0} \mathcal{J}_i(\mathbf{x}_i(\mathbf{w}_i), \mathbf{u}_i).$$

**Remark 7.** A tractable formulation for DS framework in (13), can be achieved by removing the robust coupling constraint (26c) from the tractable problem (26). Notice that, since there is no longer a coupling constraint, each agent  $i$  can solve its problem, separately.

**Remark 8.** The solution of (26) is the optimal planned input sequence  $\{u_{i,k}^*, v_{i,k}^*, \dots, u_{i,k+N_h}^*, v_{i,k+N_h}^*\}_{i=1}^N$ . Based on an MPC paradigm, the current input is implemented as  $\{u_{i,t}, v_{i,t}\}_{i=1}^N := \{u_{i,k}^*, v_{i,k}^*\}_{i=1}^N$  and we proceed in a receding horizon fashion. This means (26) is solved at each step  $t$  by using the current measurement of the state  $\{x_{i,k}\}_{i=1}^N$ .

## 5. NUMERICAL STUDY

In this section, we present a simulation study for the case study of three-agent ATES systems in a STG, as it is shown in Fig. 5. We determine the thermal energy demands of three buildings modeled using realistic parameters, that had been equipped with ATES systems, using the actual registered weather data in the city center of Utrecht, the Netherlands, where these buildings are located.

### 5.1. Simulation Setup

We simulate three problem formulations, namely: DS, CS, and MCS, using the tractable framework (26). The simulation time is one year with hourly-based sampling time. The prediction horizon for DS and CS is considered to be a day-ahead (24 hours), whereas for MCS a whole season (3 months). The control variables in MCS are considered, as the multi-rate actions, to be hourly-based during first day, daily-based in the first week, weekly-based within the first month, and monthly-based in the rest of the season. For comparison purposes, we also simulate a deterministic DS (DDS), where we fixed the uncertain elements ( $\mathbf{w}_i$ ) to their forecast value for each agent  $i = 1, 2, 3$ .

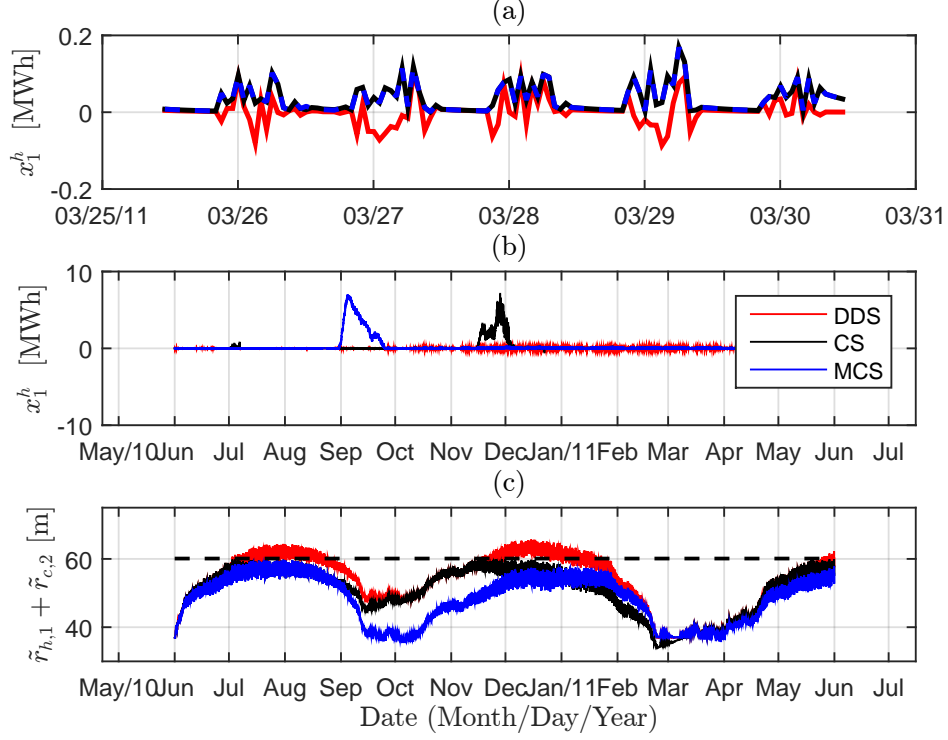


FIGURE 6. A-posteriori feasibility validation of the obtained results via DDS, DS, CS, and MCS formulations for the three-agent ATEs-STG example.

In order to generate random scenarios from the private uncertainty sources, we use a discrete normal stochastic process, where the thermal energy demand of each building varies within 10% of its actual value at each sampling time. A similar technique is used for the common uncertainty sources. The simulation environment was MATLAB with YALMIP as the interface [24] and Gurobi as the solver.

## 5.2. Simulation Results

Fig. 6 depicts an a-posteriori feasibility validation of the obtained results via DDS, DS, CS, and MCS formulations for the three-agent ATEs-STG example. Fig. 6 (a)-(b) present the results of thermal energy imbalance during heating mode in agent 1, whereas Fig. 6 (c) shows the feasibility of the coupling constraint between agent 1 and 2. Fig. 6 (a) shows the obtained results for the last five days in March 2011, and Fig. 6 (b) shows the results for one year simulation from June 2010 until June 2011. In Fig. 6 (a)-(b) the "red" color denotes the solution of DDS, "black" color shows the solution of CS, and "blue" presents the solution of MCS.

Fig. 6 (a) focuses on a five-day period to allow a better comparison between the results of DDS, CS, and MCS. It is clearly shown that the obtained results via CS and MCS, provide a feasible (nonnegative) trajectory of the thermal energy imbalance error during heating mode, whereas the solution of DDS, leads to some violations throughout the simulation time. In Fig. 6 (b), the complete one year results of DDS, CS, and MCS are shown. Two important observations are as follows: the obtained results of CS and MCS have very small number of violations, much less than our desired level of violations, throughout the simulation time. This yields a less conservative approach compared to the classical robust control approach (see [16, Ch.14]). As the second observation, in the results of CS and MCS one can see a large non-zero imbalance error, which

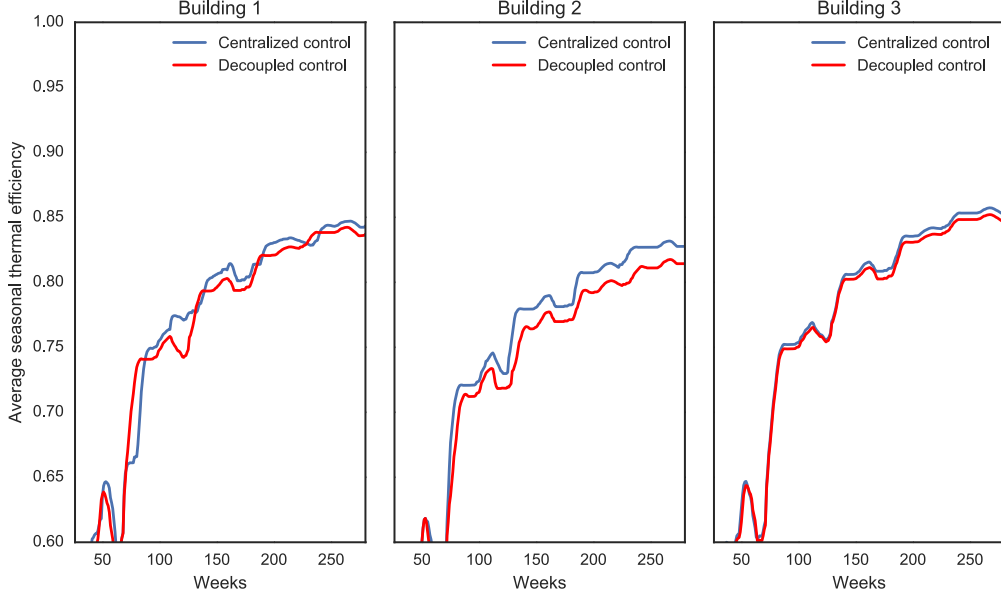


FIGURE 7. Impact of DS and CS on average thermal efficiency.

is expected. By taking into account the coupling constraints between agents, the solutions of agents are going to extract the stored thermal energy from their ATEs systems to prevent the mutual interactions between their ATEs systems as in Fig. 6 (c). Interestingly, the results of MCS shows that agent 1 starts to extract the stored thermal energy from its ATEs system sooner, compared to CS. Fig. 6 (c) shows the evaluation of our proposed reformulation in (16). We plot the obtained  $\tilde{r}_{h,1} + \tilde{r}_{c,2}$  using DS, CS, and MCS formulations. As it is clearly shown DS results are violating the coupling constraint which leads to overlap between the stored water in warm well of ATEs system in agent 1 and the stored water in cold well of ATEs system in agent 2.

It is worth to mention that Fig. 6 illustrates our three main contributions: 1) having a probabilistic feasible solution for each agent w.r.t. the private uncertainty sources as it is encoded via (22b), and 2) respecting almost surely the common resource pool between neighboring agents in STGs as it is formulated in (22c); the first and second outcomes are the direct results of our theoretical guarantee in Theorem 1. 3) prediction using a longer horizon which yields an anticipatory control decision that improves the operation of an ATEs system. This is direct consequence of our proposed move-blocking scheme in (21).

Fig. 7 shows the results of a simulation study using a more realistic aquifer simulation environment [25, MODFLOW] to validate our developed framework. Fig. 7 is the results that we obtained by integrating our control strategy, DS and CS, into Python to build a live-link with MODFLOW. The impact of our control strategy, DS (red) and CS (blue), on average thermal energy efficiency in each building is shown in Fig. 7, which illustrates that we can store and retrieve the same amount of thermal energy in ATEs systems, in a more efficient way using the results of CS formulation compared to DS. This is due to the fact that the mutual interactions between wells lead to the loss of stored thermal energy, which can be prevented using the CS formulation.

## 6. CONCLUSIONS AND FUTURE WORK

This paper proposed a stochastic MPC framework for an energy management problem in STGs consisting of ATEs systems integrated into building climate comfort systems. We developed a large-scale stochastic hybrid

model to capture thermal energy imbalance errors in an ATES-STG. In such a framework, we formalized two important practical concerns, namely: 1) the balance between extraction and injection of energy from and into the aquifers within a certain period of time; 2) the unwanted mutual interaction between ATES systems in STGs. Using our developed model, we formulated a finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints. To solve such a problem, we proposed a tractable formulation based on the so-called robust randomized approach. In particular, we extended this approach to handle a problem with multiple chance constraints. We simulated our proposed framework using a three-agent ATES-STG example which confirmed the expected performance improvements.

Our current work focuses on two main directions: 1) From an application point of view, refining the proposed model of ATES system (1) to be able to predict a more implicative situation e.g., the wells are completely depleted, or the new installed wells, and etc. Such a situation happens in a real case with a private owner, where they continue to extract water with the aquifer ambient temperature. 2) From a theoretical point of view, developing a distributed setting to solve the tractable formulation (26), e.g., extending the work in [26] to the case where binary variables are also present.

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